HIGHER MATHEMATICS PART 3.
Series. Theory of the Functions of a Complex Variable.
SYLLABUS

Description of the Discipline

Level of HE	*The first bachelor*
Branch of Knowledge	*13 Mechanical engineering*
Specialty	*131 "Applied Mechanics"*
Educational Program	*Manufacturing Engineering*
Status of the Discipline	*Compulsory.*
Form of Study	*Full-time*
Year and Semester	*Second year, autumn semester*
Number of Credits	*4 credits ECTS (120 hours), including 36 hours of Lectures, 36 hours pf Practical Lessons, 48 hours of Self-study work*
Semester's Control	*Exam/ Midterm test, Individual tasks*
Schedule	*According to the schedule on the university website*
Language	*English*
Information About Course Teacher	*Ganna Zhuravska, Ph.D., Associate Professor* https://mph.kpi.ua/osobovij-sklad.html
Learning Resources	*Learning resources are determined by the course teacher and are given to students in on the first lesson. They consist of links to remote resources in Moodle, Google classroom, information resources in the university library, on the website of the department, etc.*

Course Description

1. Learning Objectives, Tasks and Outcomes of the Discipline

The discipline "Higher Mathematics. Part 3. Series. Theory of Functions of a Complex Variable" is the third and final part of the compulsory component "Higher Mathematics" within the cycle of professional training for bachelor students in specialty G9 "Applied Mechanics."

The aim of the course is to develop students' ability to master the fundamental concepts and methods of differential and integral calculus of functions of several variables, multiple, line and surface integrals, elements of field theory, theory of numerical and functional series; to use theoretical material to solve typical problems; to apply acquired knowledge and skills to practical problems in mathematics, mechanics, physics and everyday professional practice; and to work independently with mathematical literature and other information sources.

According to the matrix of correspondence of program competencies

- *general competencies:*

GC1. Ability for abstract thinking, analysis, and synthesis;

- *professional competencies:*

PC1. Ability to analyze materials, structures, and processes based on the laws, theories, and methods of mathematics, natural sciences, and applied mechanics.

Students must demonstrate the following learning outcomes.

According to the matrix of correspondence of program learning outcomes in the educational program

- PLO1. Select and apply appropriate mathematical methods to solve problems in applied mechanics.

Knowledge: the bases of the integral calculus of functions of one and many variables (indefinite integral, properties of an indefinite integral, a table of integrals, methods of integration); the bases of the integral calculus of functions of several variables (definite integral and its geometric interpretation, fundamental theorem of calculus, improper integrals, double and triple integrals, methods of calculation in different coordinate systems, line and surface integrals, application of all kinds of integrals to problems of geometry and physics, vector calculus);the bases of the theory of series (concept of numerical series, convergence and divergence of a series, absolute and conditional convergence for alternating series, function series, power series, interval of convergence, Taylor series, Fourier series, applications of series); the fundamentals of the theory of functions of a complex variable and an operational calculus (concepts, derivatives, and integrals, Taylor series and Laurent series of functions of a complex variable, Laplace and Fourier transformations, application of the Laplace transformation for solving ordinary differential equations and systems of such equations).

Skills: to find partial derivatives and differentials for a function of several variables; to apply partial derivatives; to find extrema of a function of two variables; to find the gradient and directional derivative of a scalar field; to find double, triple, line and surface integrals in different systems of coordinates, to apply all kinds of integrals; to study numerical, functional and power series, to use Taylor series and Fourier series, to orientate in the fields of their application; to find derivative and to calculate integrals of a functions of a complex variable, to represent a function in the form of Taylor or Laurent series; to use Laplace transforms for solving ordinary differential equations and systems of such equations.

Experience: thinking logically and flexibly; independently using and studying literature on mathematics; generalizing, setting the goal and choosing ways to solve it; using methods of mathematics in engineering calculations.

2. Prerequisites and Postrequisites of the Discipline (place in the structural and logical scheme of education according to the relevant educational program)

The course *"Higher Mathematics. Part 3. Series. Theory of Functions of a Complex Variable"* is taught in the third semester, based on the previous two parts: *"Higher Mathematics. Part 1. Differential and Integral Calculus of Functions of One Variable"* and *"Higher Mathematics. Part 2. Differential and Integral Calculus of Functions of Several Variables. Differential Equations"*, which were studied in the preceding two semesters. This discipline supports subsequent courses such as *"Theoretical Mechanics"*, *"Theoretical Foundations of Heat Engineering"*, *"Fluid and Gas Mechanics"*, and others according to the structural-logical scheme of the respective educational program.

3. Content of the Discipline

Section 1. Differential Calculus of Functions of Several Variables

Topic 1.1. Function of several variables and its properties.

Topic 1.2. Partial derivatives and differentials of functions of several variables.

Topic 1.3. Applications of partial derivatives of functions of several variables.

Section 2. Multiple Integrals

Topic 2.1. Double integral. Change of variables in a double integral.

Topic 2.2. Applications of the double integral to problems in geometry and mechanics.
Topic 2.3. Triple integral. Change of variables in a triple integral.
Topic 2.4. Applications of the triple integral to problems in geometry and mechanics.

Section 3. Line and Surface Integrals of the First Kind

Topic 3.1. Line integrals of the first kind (arc length).
Topic 3.2. Surface integrals of the first kind.

Section 4. Elements of Field Theory

Topic 4.1. Scalar and vector fields. Differential characteristics of fields.
Topic 4.2. Line integral of the second kind (by coordinates).
Topic 4.3. Surface integral of the second kind.

Section 5. Numerical Series

Topic 5.1. Positive term numerical series.
Topic 5.2. Alternating series.

Section 6. Functional Series

Topic 6.1. Functional series. Uniform convergence of a functional series.
Topic 6.2. Power series. Taylor and Maclaurin series.
Topic 6.3. Applications of power series.

Section 7. Fourier Series

Topic 7.1. Fourier series for a 2π-periodic function.
Topic 7.2. Fourier series for a 2l-periodic function.
Topic 7.3. Fourier series for functions defined on the interval [0,l] or [a,b].

4. Informational Support

Readings

1. N.Piscunov Differential and Integral Calculus/ N.Piscunov – Mir Publisher, Moscow, 1966 – 895 p.
2. Ganna Zhuravska Higher Mathematics. Series. Elements of Theory [Electronic resource] / Ganna Zhuravska. – Electronic text data (1 file: 1,3 MB). – Kyiv : Igor Sikorsky Kyiv Polytechnic Institute, 2021. – 67 p. https://ela.kpi.ua/handle/123456789/41247
3. Theory of Series Elements of Theory of Functions of a Complex Variable Operational Calculus [Electronic resource] : textbook/ Tamara Karpaluk, Inna Kopas, Nadia Reva, Ganna Zhuravska ; Igor Sikorsky Kyiv Polytechnic institute. – Electronic text data (1 file: 5,33 Mb). – Kyiv : Igor Sikorsky Kyiv Polytechnic Institute, 2024. – 316 p. https://ela.kpi.ua/handle/123456789/67151
4. Higher Mathematics. Multivariable Calculus. Vector Calculus. Elements of Theory [Electronic resource] / Igor Sikorsky Kyiv Polytechnic Institute ; compiler: Ganna Zhuravska. – Electronic text data (1 file: 3,13 MB). – Kyiv : Igor Sikorsky Kyiv Polytechnic Institute, 2021. – 110 p. https://ela.kpi.ua/handle/123456789/41246
5. Swokowski, Earl William Calculus: 5th ed / Swokowski, Earl William - Published 1991 by Pws-Kent Publishing Company – ISBN 0-534-92492-1 - 1053 p.

Educational Content

5. Learning Activities and Teaching Methods

Basic teaching methods:

1. Providing during lectures theoretical material and examples of solving the main practical problems.

2. Solving examples and tasks during practical trainings through active and collective learning, part-search and research methods;

3. Self-studying of students, such as homework to each topic of the course and individual tasks.

4. As an addition to traditional training activities there are proposed such activities as internet mailing, distance and online learning, online testing etc.

5. Assessment is carried out with the help of various control measures: midterm tests, mathematical dictation, classroom activities, homework, individual tasks and final exam.

Lectures

№	The title of the lecture and a list of the main subtopics
1	Function of several variables and its properties. Partial derivatives and differentiability of functions of several variables. Differential of a function of several variables.
2	Applications of partial derivatives. Tangent plane and normal to a surface. Local extrema of a function of two variables. Maximum and minimum values of a function of two variables.
3	Double integral and its properties. Evaluation of the double integral in Cartesian coordinates.
4	Change of variables in a double integral. Double integral in polar coordinates. Applications of the double integral to problems in geometry and mechanics.
5	Triple integral and its properties. Evaluation of the triple integral in Cartesian coordinates.
6	Change of variables in a triple integral. Cylindrical and spherical coordinate systems. Evaluation of the triple integral in cylindrical and spherical coordinates. Applications of the triple integral to problems in geometry and mechanics.
7	Line integral of the first kind (arc length). Evaluation and applications of line integrals of the first kind. Surface integral of the first kind. Evaluation and applications of surface integrals of the first kind.
8	Basic concepts of field theory. Scalar field. Directional derivative. Gradient of a scalar field and its properties. Vector field. Concepts of divergence and curl of a vector field.
9	Line integral of the second kind (by coordinates). Evaluation and applications of line integrals of the second kind. Circulation of a vector field. Green’s theorem. Conditions for independence of a line integral from the path of integration.
10	Surface integrals of the second kind. Evaluation and applications of surface integrals of the second kind. Flux of a vector field. Stokes’ theorem. Gauss–Ostrogradsky theorem.
11	Numerical series. Convergence of a numerical series. Necessary condition for convergence of a numerical series. Positive term series. Sufficient conditions for convergence.

12	Alternating series. Leibniz criterion. Absolute and conditional convergence. Power series and their convergence. Properties of power series.
13	Taylor and Maclaurin series. Expansion of elementary functions into a Maclaurin series. Applications of power series.
14	Trigonometric Fourier series. Fourier series for even and odd functions.
15	Fourier series for a 2*l*-periodic function. Fourier series for functions defined on the interval [0,*l*] or [*a*,*b*].

Practical Trainings

№	The title of the practical training
1	Function of several variables and its properties. Partial derivatives and total differentials of the 1st and 2nd order.
2	Applications of partial derivatives. Tangent plane and normal to a surface. Local extrema of a function of two variables. Maximum and minimum values of a function of two variables.
3	Double integral. Evaluation of the double integral in Cartesian coordinates. Applications to problems in geometry and mechanics.
4	Change of variables in a double integral. Double integral in polar coordinates. Applications to problems in geometry and mechanics.
5	Triple integral. Evaluation of the triple integral in Cartesian coordinates.
6	Change of variables in a triple integral. Evaluation of the triple integral in cylindrical and spherical coordinates.
7	Applications to problems in geometry and mechanics.
8	Line and surface integrals of the first kind. Properties and evaluation.
9	Scalar field. Directional derivative. Gradient of a scalar field.
10	Vector field. Concepts of divergence and curl.
11	Line integral of the second kind. Green’s theorem. Integration of exact differentials.
12	Surface integrals of the second kind. Flux of a vector field.
13	Gauss–Ostrogradsky theorem. Stokes’ theorem. Circulation of a vector field.
14-15	TEST – 1 (45 minutes). Numerical series. Necessary condition for convergence of a series. Positive term series. Comparison tests.
16	D’Alembert’s test, Cauchy’s root test, Cauchy’s integral test.
17	Alternating series. Leibniz criterion for convergence of alternating series. Absolute and conditional convergence.
18	Power series. Interval and radius of convergence of a power series.
19	Taylor and Maclaurin series. Expansion of elementary functions into a Maclaurin series.
20	Applications of power series.
21-22	Trigonometric Fourier series. Expansion of periodic functions into a Fourier series. TEST – 2 (45 minutes).

6. Self-studying of Students

A total of **76 hours** of study time is allocated for independent student work.
It includes:

- Reviewing lectures, preparing for classroom sessions, completing homework assignments and tests — **27 hours**.
- Preparing for control works — **4 hours**.
- Completing tasks of the calculation assignment — **15 hours**.
- Preparing for the exam — **30 hours**.

Policies and Assessments

7. Policies of the Discipline

Class-meetings are held in auditoriums according to the schedule. In the case of online classes that should be provided by the relevant order of the university, lectures and practical trainings are engaged in online video communications (Zoom, Google-Meet, Skype).

Requirements and the system of assessment are announced to students in the first lesson.

Attendance

Students are expected to attend each class meeting, since during these lessons they study theoretical material and develop the skills needed to complete homeworks, individual tasks and middle tests. There are no penalty points for absence from lectures and practical classes.

Absence During Control

Failure to submit the scheduled time the individual tasks and systematic failure to do homeworks without a valid reason is punishable by penalty points, according to the Rating System. Absence from the middle test, if the reason is not documented, is not give you a chance to do it in another time.

Academic Integrity Policy

The policy and principles of academic integrity are defined in Section 3 of the Code of Honor of the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute".

Read more: https://kpi.ua/code.

Norms of Ethical Behavior

Norms of ethical behavior of students and teachers are defined in Section 2 of the Code of Honor of the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute".

Read more: https://kpi.ua/code.

8. Assessment of Learning Outcomes and Rating System

Execution ongoing control and assessment of student's knowledge are made due to classroom activities, middle tests and individual tasks, the purpose of which is to identify the level of assimilation of relevant topics.

The final test is an exam in the end of each semester. The 100-points rating system and the university scale are used to evaluate the results of the training.

The Rating System

1. The student rating of the credit module is calculated from 100 points, 50 of them are the starting scores. Starting scores (during the semester) consists of the points that the student receives for:

1) classroom activities (22 lessons);
2) homework;
3) two midterm tests;
4) individual tasks.

2. Criteria For Scoring Points

<u>2.1. Classroom Activities:</u>

– student has a thorough knowledge and understanding of the content and a high level of competence in the processes and skills - 1 points;

– student has achieved a basic level of competence in the processes and skills – 0.5 point;

– student has no elementary knowledge and understanding of the content - 0 points.

The maximum of points for classroom activities during the semester is 5.

<u>2.2. Homework</u>

It includes completing tasks at home for each topic covered. Correctly completed and timely homework is assessed at 0.25 points.

The maximum of points for all homework is 0.25 ×20=5 points.

<u>2.3. Midterm Test</u>

There are two Midterm tests (for 10 points each)

- complete solutions (no less than 90% of the required information) –9-10 points;
- solutions with minor inaccuracies (no less than 75% of the required information) – 7.5-8.5 points;
- mistakes in an solutions (no less than 60% of the required information) - 6-7 points;
- no solutions - 0.

Module control works and all other written assignments must be completed by students independently, without the use of auxiliary materials (reference books, mobile phones, tablets, etc.). To confirm the fact of independent completion of any written work, the student must be able to orally demonstrate full understanding of the completed control work.

In the case of distance learning, a control work that was supposed to be conducted in the classroom is written by students during practical classes according to the schedule, using platforms such as Zoom, Google Meet, or others as agreed with the lecturer. Students are sent the tasks for the control work, and within the allotted time for writing, they must submit their completed solutions. If the work is not submitted on time, it is considered that the student was absent from the control work, the work is not checked, and the student receives **0 points** (unless there is a valid reason).

As an exception, for a valid reason, the control work may be rescheduled to another day (by prior agreement with the lecturer).

Repeated writing of a module control work is **not allowed**.

<u>2.4. Individual Tasks - 20 points.</u>

- complete solutions – 18-20 points;
- solutions with minor inaccuracies - 15-17 points;
- mistakes in an solutions - 12-14 points;
- no solutions - 0 points.

The student must submit the individual tasks of the calculation assignment on time for review; the deadlines for submitting parts of the calculation assignment are determined by the lecturer. The fully completed calculation assignment must be submitted no later than the last class of the semester. If this deadline is violated, the student is considered **not admitted** to the exam of the main session.

Subsequently, to obtain admission to the exam of the additional session, the student may submit and defend the calculation assignment only for the minimum passing grade, which is 60% of the maximum possible points for the calculation assignment.

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n the case of distance learning, the completed calculation assignment must be sent to the lecturer's email or another platform agreed upon with the lecturer.

The condition for a positive mid-term (calendar) assessment is that the student obtains at least **50% of the maximum possible points** at the time of assessment.

The condition for admission to the exam is a credited calculation assignment and an individual semester rating of at least **60% of the maximum possible semester points (50 points)**, i.e., **30 points**.

Final Exam

During the exam, students complete an **examination control work**. Each exam paper contains **two theoretical questions** and **three practical tasks**, each evaluated on a 10-point scale according to the following criteria:

System for evaluating theoretical questions:

Excellent: complete answer (at least 90% of required information) — **9–10 points**.
Good: sufficiently complete answer (at least 75% of required information), or minor inaccuracies — **7.5–8 points**.
Satisfactory: incomplete answer (at least 60% of required information) with some mistakes — **6–7 points**.
Unsatisfactory: inadequate answer — **0–5 points**.

System for evaluating practical tasks:

Excellent: complete and error-free solution — **9–10 points**.
Good: complete solution with minor inaccuracies — **7.5–8 points**.
Satisfactory: solution completed with certain shortcomings — **6–7 points**.
Unsatisfactory: task not completed — **0–5 points**.

During the exam, the use of any reference materials, mobile phones, or other devices is strictly prohibited.

Final Grade Calculation The sum of the semester points and the points earned for the examination control work is converted into the final exam grade according to the **university rating scale correspondence table**.

PERSONAL SCORE	GRADE
95-100	Excellent
85-94	Very Good
75-84	Good
65-74	Satisfactory
60-64	Sufficiently
Less than 60	Unsatisfactory
Less than 36 or the individual tasks is not credited	Not allowed to pass exam

Final Exam in Distance Learning

In the case of distance learning, by decision of the university administration, there is a possibility of awarding the exam grade *"automatically"* (with the student's consent) through recalculation of the semester points according to the 100-point scale:

$$R = 60 + \frac{40(R_I - R_D)}{R_C - R_D}$$

where:

- R_C=50 points — maximum sum of weighted points for control activities during the semester,
- R_I — sum of points earned by the student during the semester (individual semester rating),
- R_D=30 points — minimum admission threshold to the exam.

If the student does not agree with the *automatic* grade, they take the exam in video-conference format according to the exam session schedule.

If the student's individual rating R_I<30 points, they are considered **not admitted** to the exam of the main session. However, if the calculation assignment is credited, the student is given the opportunity to obtain admission to the exam of the additional session by completing additional control activities.

Syllabus of the Discipline:

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