Conditions for the finite speed of propagation of non-negative solutions of the Bussinesque equation with nonhomogeneous density

V.A. Vasylenko, V.M. Shramenko
Faculty of Physics and Mathematics of
the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute"
e-mail: vasvlad1997@gmail.com

We examine problem in fluid mechanics. It deals with the filtration of an incompressible fluid (typically, water) through a porous stratum, the main problem in groundwater infiltration. The model was develobed first by Boussinesq in 1903. [1]

For this model we have Boussinesq's equation

$$\rho\frac{\partial u}{\partial t} = \Delta u^2$$

We know that solution of equation (1) has the property of a finite speed of propagation of perturbations, when $\rho = const$.

We assume that $\rho = (1 + |x|)^{-l}$, $l > 0$, and consider the Cauchy problem

$$\rho\frac{\partial u}{\partial t} = \Delta u^2 \tag{1}$$

in $Q_T = \mathbb{R}^3 \times (0, T)$,

$$u(x, 0) = u_0(x), \quad x \in \mathbb{R}^3, \quad u_0(x) \geq 0 \quad \text{for a.e.} \quad x \in \mathbb{R}^3. \tag{2}$$

There $u = u(x, t), \quad x = (x_1, x_2, x_3), \quad |x| = (x_1^2 + x_2^2 + x_3^2)^{\frac{1}{2}}$.

In addition

$$\sup u_0 \subset B_{R_0} \equiv \{|x| < R_0\}, \quad \|u_0\|_{\infty,\mathbb{R}^3} < \infty.$$

We say that the solution of the equation (1) has a property of a finite speed of propagation of perturbations if from the condition $\sup(\cdot, t) < \infty$ at some point in time $t_0 \geq 0$ it follows that this property is preserved for all moment times $t \geq t_0$.

Let us formulate the main result of this work.

Theorem. *Let $u(x, t)$ – a solution of the problem (1),(2) in Q_T. If $0 < l < \frac{5}{2}$, then $u(x, t)$ has a property of a finite speed of propagation of perturbations.*

References

[1] Juan Luis Vazquez. The Porous Medium Equation, 2007

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